How could Hawking Radiation can be obtained. Loop corrections

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June 28, 2015

Outline

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2. Gravitational preliminary
3. In-harmonics of free scalar field
4. Hawking radiation
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Setup of the problem

We consider a massive scalar field on a thin-shell collapse background in 4D.

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \lambda\phi^4 - \dots$$

$$ds^2 = \begin{cases} ds_+^2 = \left(1 - \frac{r_g}{r}\right) dt_+^2 - \frac{dr^2}{1-\frac{r_g}{r}} - r^2 d\Omega_2^2, r > R(t) \\ ds_-^2 = dt_-^2 - dr^2 - r^2 d\Omega_2^2, r < R(t) \end{cases}$$

Where $R(t)$ is a trajectory of shell. It is assumed, that initially the shell didn't move and was at radius R_0 and after some moment it starts to move.

$$R(t) = \begin{cases} R_0, t \leq 0 \\ R(t), t > 0 \end{cases}$$

We want to understand <u>what is happening with scalar field during collapse</u>

We have to sew metric inside and outside the shell. Before collapse appeared it leads to the equality

$$ds^2_{sh,-} = ds^2_{sh,+} \Rightarrow t_- = \sqrt{1 - \frac{r_g}{R_0}} t_+$$

The thin-shell collapses according to the following law(that is obtained by using Einstein equation)

$$R(t_+) \approx r_g + \frac{R_0 - r_g}{v} e^{-\frac{t_+}{r_g}}, R(t_-) = R_0 - vt$$

And we get how to express t_- through t_+

$$t_- \approx \frac{R_0 - r_g}{v} \left(1 - e^{-\frac{t_+}{r_g}}\right)$$

EOM for free harmonics

The simplest case is a free scalar field. Also we consider only in-harmonics, that diagonalize Hamiltonian, when shell doesn't move.

EOMs for harmonics are

$$\phi(t,r,\theta,\alpha)=\sum_{l,m}Y_{l,m}(\theta,\alpha)\phi_{l,m}(r,t)$$

$$\begin{cases}\left[\partial_{t_-}^2-\partial_r^2+m^2+\frac{l(l+1)}{r^2}\right](r\phi_l)=0, r<R(t)\\ \left[\partial_{t_+}^2-\partial_{r^*}^2+(1-\frac{r_g}{r})\left(m^2+\frac{l(l+1)}{r^2}+\frac{r_g}{r^3}\right)\right](r\phi_l)=0, r>R(t)\end{cases}$$

$$r^*=r+r_g\log\left(\frac{r}{r_g}-1\right)$$

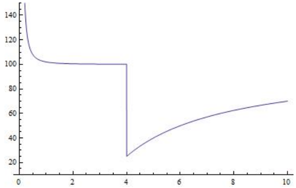
$$V(r_*)=\begin{cases}\left(1-\frac{r_g}{r_*}\right)\left(m^2+\frac{l(l+1)}{r_*^2}+\frac{r_g}{r_*^3}\right), r<R\\ m^2+\frac{l(l+1)}{r_*^2}, r>R\end{cases}$$

Spectrum of the in-theory

In order to find a spectrum we consider the following dependence on t_+ for harmonics

$$\phi_l \propto e^{-i\omega t} \Rightarrow \left[-\partial^2_{r_*} + V(r^*)\right] \phi_l = \omega^2 \phi_l$$

It reminds Schroedinger equation and the spectrum is the energy states for the following potential



We conclude that spectrum of theory is

1. Discrete in the range $m\left(1 - \frac{r_g}{R_0}\right) < \omega < m$
2. Continuous for $\omega > m$

The further evolution of harmonics

Under thin-shell, free field doesn't know about collapse and hence it depends on time as the follow

$$\phi_l \propto e^{-i\omega t_-} \Rightarrow \phi_l(R, t_+) \propto e^{-i\omega \frac{R_0 - r_g}{v} e^{-\frac{t_+}{r_g}}}$$

We assume, that $v \approx 1$. Near horizon the general solution of free field equation is

$$\phi_l = f(\underbrace{t + r_*}_{u}) + g(\underbrace{t - r_*}_{v})$$

By imposing the condition of continuity we get harmonics outside of shell

$$\phi_l(u, v) = \frac{1}{\sqrt{\omega r_g}} e^{i\omega r_g e^{-\frac{u}{2r_g}}} + \frac{1}{\sqrt{2\omega r_g}} e^{-i\omega v}$$

Let us calculate fluxes of J_u and J_v for stationary observer

$$J_u = \int r^2 d\Omega_2 \left(\partial_u \phi\right)^2, J_v = \int r^2 d\Omega_2 \left(\partial_v \phi\right)^2$$

Straightforward calculation shows that

$$J_v = \int \frac{d\omega}{2\pi}\frac{\omega}{2}, J_u = \int \frac{d\omega}{2\pi}\frac{\omega}{2} + \int \frac{d\omega}{2\pi}\omega n(\omega), n(\omega) = \frac{1}{e^{4\pi r_g \omega} - 1}$$

And total flux is a "thermal" one.

$$J_{st.} = \int \frac{d\omega}{2\pi}\frac{\omega}{e^{4\pi r_g \omega} - 1}$$

Loop corrections

By using such harmonics we can calculate loop corrections to some quantity. But because situation is non-stationary we have to use Schwinger Keldysh formalism.

It contains three propagators $D^{K,R,A}$.

The most interesting one is D^K because it contains information about level population $D^K \propto f_\omega f_{\omega'} n_{\omega,\omega'}$, $n_{\omega,\omega'} = \langle a^\dagger_\omega a_{\omega'} \rangle$. One-loop perturbation level gives

$$n_{\omega,\omega'}(t) = \lambda^2 \int_{t_1 \leq t} d^4x_1 \int_{t_2 \leq t} d^4x_2 f_\omega(x_1) f^*_{\omega'}(x_2) \prod_{i=1}^{3} \int \frac{d\omega_i}{2\pi} f_{\omega_i}(x_1) f^*_{\omega_i}(x_2)$$

$$n_{\omega,\omega'}(t) \propto \lambda^2 (t - t_0)$$

Where t_0 is a moment when shell starts to move. Such linear growth corresponds to particle creation.

Discussion

To understand the meaning of it we have to sum up the leading contributions from all level of perturbation series.
It can lead to the following possibilities.

1. The summation can lead to the constant shift of the number of particles.
2. The "explosion", the number of particles at some moment will be equal to infinity and thin shell is destroyed.

It modifies Hawking radiation and can be crucial for the understanding of Information Paradox.

Conclusion

1. The spectrum of scalar field theory was considered in thin-shell metric background.
2. The flux that is seen by falling and stationary observers was obtained.
3. Linearly growing with time loop corrections was obtained for scalar field in gravitational collapse background.

Thank you for your attention!